KINETICS OF THE TEMPERATURE FIELD INDUCED BY THE EMISSION OF A LASER WITH TIME-VARYING INTENSITY

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The kinetics of the temperature field when a medium is heated with a laser beam with a Gaussian intensity distribution over the radius and different time dependences are obtained. Numerical estimates are given.

Laser radiation propagating through an optical medium heats it and changes its refractive index. This affects the phase front of the laser radiation and may lead to self-focusing of the laser beam. The thermal field gives rise to thermal stresses which may lead to destruction of optical components. For a more accurate understanding of the processes brought about by the heating of the medium it is necessary to know the value and distribution of the temperature field induced by the laser radiation. The problem of the heating of a weakly absorbing medium by laser radiation has been considered in [1-3]. In a semiinfinite medium the kinetics of the temperature field produced by laser radiation whose intensity is constant with time, when the heated region has the form of a cylinder, has been considered. Values for the thermal field during the time the laser radiation acts were obtained. To determine the kinetics of the variation of the refractive index, the thermal stresses, and also to choose the shape of the laser radiation pulse that is optimum from the point of view of radiation resistance, it is of interest to know the value of the temperature field not only during the laser pulse, but also after the pulse is completed, and it is also of interest to take into account the variation of the laser radiation intensity with time when calculating the temperature field. In the present paper expressions are obtained for the temperature field during the laser pulse and after it is completed; the calculations are carried out for laser radiation that is constant with time and for an intensity which varies linearly, quadratically, and exponentially with time.

To solve the thermal-conduction problem, analytical calculations were carried out for the following conditions and limitations.

1. A weakly absorbing isotropic medium ($kl \ll 1$, where k is the linear absorption coefficient and l is the length of the specimen) or a uniaxial crystal, whose optic axis coincides with the direction of propagation of the laser radiation, is considered.

2. The medium has the form of an infinite plate. The direction of the radiation is perpendicular to its surface. There is no heat exchange on the surfaces of the plate.

3. The intensity distribution of the laser radiation over the cross section of the beam is axisymmetrical (Gaussian), and the heated region has the form of a cylinder.

4. It is assumed that the initial temperature of the specimen and the temperature at infinity are zero.

Consequently, assuming that under the conditions considered the temperature field is axisymmetrical and is independent of the coordinate z, the problem will be solved in a cylindrical system of coordinates.

Solution of the Thermal-Conduction Equation

The temperature field produced by absorption of the laser emission is found by solving the thermal-conduction equation

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$$\lambda\nabla^2 T(r,\ t) + q = c\rho\frac{\partial T(r,\ t)}{\partial t}, \tag{1}$$

where T is the temperature field; λ is the thermal conductivity; c and ρ are the specific heat and density of the medium, respectively; $q = kI$ is a volume source of heat; k is the linear light absorption coefficient; and $I(r,\ t)$ is the intensity of the laser radiation.

Assuming that the intensity of the laser radiation can be written in the form $I(r,\ t) = I(t)f(r)$, the solution of Eq. (1) with the boundary conditions

$$T(r,\ 0) = 0,\ T(\infty,\ t) = 0 \tag{2}$$

will be

$$T(r,\ t) = \int_0^t I(t')\, T_1(r,\ t - t')\, dt'. \tag{3}$$

Here $T_1(r,\ t)$ is the solution of the equation

$$\lambda\nabla^2 T_1(r,\ t) = c\rho\frac{\partial T_1(r,\ t)}{\partial t}$$

with the boundary conditions

$$T_1(r,\ 0) = \frac{k}{c\rho} f(r),\ T_1(\infty,\ t) = 0$$

where $T_1(r,\ t)$ denotes the temperature field which is produced by relaxation after instantaneous heating of the medium to a temperature $(k/c\rho)f(r)$.

For a Gaussian beam $[f(r) = \exp(-r^2/r_0^2)]$, T_1 has the form (see, for example, [3])

$$T_1(r,\ t) = \frac{k}{c\rho}\cdot\frac{1}{1+\frac{t}{\tau}}\exp\left(-\frac{r^2}{r_0^2}\cdot\frac{1}{1+\frac{t}{\tau}}\right), \tag{4}$$

where $\tau = r_0^2 c\rho/4\lambda$ is the characteristic time of departure of the temperature from the heated region.

Using Eqs. (3) and (4) we will obtain the values of the temperature field when a medium is heated by laser radiation with an intensity that is constant with time and which varies linearly, quadratically, and exponentially with time.

a) A pulse of laser radiation, rectangular in time:

$$I(t) = \begin{cases} 0, & t < 0, \\ I_0, & 0 \leqslant t \leqslant t_0, \\ 0, & t > t_0. \end{cases}$$

Taking Eq. (4) into account we obtain from Eq. (3) the temperature field

$$T(r,\ t) = \begin{cases} 0, & t < 0, \\ \frac{kI_0\tau}{c\rho}\left[\mathrm{Ei}\left(-\frac{r^2}{r_0^2}\right) - \mathrm{Ei}\left(-\frac{r^2}{r_0^2}\cdot\frac{\tau}{\tau+t}\right)\right], & 0 \leqslant t \leqslant t_0, \\ \frac{kI_0\tau}{c\rho}\left[\mathrm{Ei}\left(-\frac{r^2}{r_0^2}\cdot\frac{\tau}{\tau+t-t_0}\right) - \mathrm{Ei}\left(-\frac{r^2}{r_0^2}\cdot\frac{\tau}{\tau+t}\right)\right], & t > t_0, \end{cases} \tag{5}$$

where

$$\mathrm{Ei}(x) = \int_{-\infty}^{x}\frac{e^t}{t}\,dt,\ x < 0.$$

The expression for the temperature on the axis of the beam takes the simpler form

$$T(0,\ t) = \begin{cases} 0, & t < 0, \\ \frac{kI_0\tau}{c\rho}\ln\left(1+\frac{t}{\tau}\right), & 0 \leqslant t \leqslant t_0, \\ \frac{kI_0\tau}{c\rho}\ln\frac{\tau+t}{\tau+t-t_0}, & t > t_0. \end{cases} \tag{6}$$

b) The intensity of the laser emission increases linearly with time:

$$I(t)=\begin{cases}0, & t<0,\\ I_0\dfrac{t}{t_0}, & 0\leqslant t\leqslant t_0,\\ 0, & t>t_0.\end{cases}$$

The temperature field in this case is given by the expression

$$T(r,\ t)=\begin{cases}0,\quad t<0,\\ \dfrac{kI_0\tau^2}{c\rho t_0}\left\{\left[\operatorname{Ei}\left(-\dfrac{r^2}{r_0^2}\right)-\operatorname{Ei}\left(-\dfrac{r^2}{r_0^2}\dfrac{\tau}{\tau+t}\right)\right]\left(1+\dfrac{r^2}{r_0^2}+\dfrac{t}{\tau}\right)+\right.\\ \left.+\exp\left(-\dfrac{r^2}{r_0^2}\right)-\left(1+\dfrac{t}{\tau}\right)\exp\left(-\dfrac{r^2}{r_0^2}\cdot\dfrac{\tau}{\tau+t}\right)\right\},\quad t_0\geqslant t\geqslant 0,\\ \dfrac{kI_0\tau^2}{c\rho t_0}\left\{\left[\operatorname{Ei}\left(-\dfrac{r^2}{r_0^2}\cdot\dfrac{\tau}{\tau+t-t_0}\right)-\operatorname{Ei}\left(-\dfrac{r^2}{r_0^2}\dfrac{\tau}{\tau+t}\right)\right]\left(1+\right.\right.\\ \left.+\dfrac{r^2}{r_0^2}+\dfrac{t}{\tau}\right)+\left(1+\dfrac{t-t_0}{\tau}\right)\exp\left(-\dfrac{r^2}{r_0^2}\cdot\dfrac{\tau}{\tau+t-t_0}\right)-\\ \left.-\left(1+\dfrac{t}{\tau}\right)\exp\left(-\dfrac{r^2}{r_0^2}\dfrac{\tau}{\tau+t}\right)\right\},\quad t>t_0.\end{cases}\tag{7}$$

The temperature on the axis of the beam varies with time as given by

$$T(0,\ t)=\begin{cases}0,\quad t<0\\ \dfrac{kI_0\tau^2}{c\rho t_0}\left[\left(1+\dfrac{t}{\tau}\right)\ln\left(1+\dfrac{t}{\tau}\right)-\dfrac{t}{\tau_0}\right],\quad 0\leqslant t\leqslant t_0,\\ \dfrac{kI_0\tau^2}{c\rho t_0^2}\left[\left(1+\dfrac{t}{\tau}\right)\ln\left(\dfrac{\tau+t}{\tau+t-t_0}\right)-\dfrac{t_0}{\tau}\right],\quad t>t_0.\end{cases}\tag{8}$$

If the intensity of the laser decreases linearly with time, the temperature field will be given by the difference between Eqs. (5), (6) and (7), (8).

c) A parabolic variation of the intensity of the laser emission with time:

$$I(t)=\begin{cases}0, & t<0,\\ 4I_0\left(-\dfrac{t^2}{t_0^2}+\dfrac{t}{t_0}\right), & t_0\geqslant t\geqslant 0,\\ 0, & t>t_0.\end{cases}$$

The solution for the temperature field has the form

$$T(r,\ t)=\begin{cases}0,\quad t<0.\\ \dfrac{4kI_0\tau^3}{c\rho t_0^2}\left\{\left[\operatorname{Ei}\left(-\dfrac{r^2}{r_0^2}\right)-\operatorname{Ei}\left(-\dfrac{r^2}{r_0^2}\cdot\dfrac{\tau}{\tau+t}\right)\right]\left[\left(1+\dfrac{t}{\tau}\right)\times\right.\right.\\ \left.\times\left(\dfrac{t_0-t}{\tau}-\dfrac{2r^2}{r_0^2}-1\right)+\dfrac{r^2}{r_0^2}\cdot\dfrac{t}{\tau}-\dfrac{r^4}{2r_0^4}\right]+\dfrac{\tau+t}{2\tau^2}\left(3\tau\ 3t-2t_0+\tau\dfrac{r^2}{r_0^2}\right)\exp\left(-\dfrac{r^2}{r_0^2}\cdot\dfrac{\tau}{\tau+t}\right)-\\ \left.-\dfrac{3\tau+4t-2t_0+\tau^2\dfrac{r^2}{r_0^2}}{2\tau}\exp\left(-\dfrac{r^2}{r_0^2}\right)\right\},\quad t_0\geqslant t\geqslant 0,\\ \dfrac{4kI_0\tau^3}{c\rho t_0^2}\left\{\left[\operatorname{Ei}\left(-\dfrac{r^2}{r_0^2}\cdot\dfrac{\tau}{\tau+t-t_0}\right)-\operatorname{Ei}\left(-\dfrac{r^2}{r_0^2}\cdot\dfrac{\tau}{\tau+t}\right)\right]\times\right.\\ \times\left[\left(1+\dfrac{t}{\tau}\right)\left(\dfrac{t_0-t}{\tau}-\dfrac{2r^2}{r_0^2}-1\right)+\dfrac{r^2}{r_0^2}\cdot\dfrac{t}{\tau}-\dfrac{r^4}{2r_0^4}\right]+\\ +\dfrac{\tau+t}{2\tau^2}\left(3\tau+3t-2t_0+\tau\dfrac{r^2}{r_0^2}\right)\exp\left(-\dfrac{r^2}{r_0^2}\cdot\dfrac{\tau}{\tau+t}\right)-\\ \left.-\dfrac{\tau+t-t_0}{2\tau^2}\left(3\tau+3t-t_0+\tau\dfrac{r^2}{r_0^2}\right)\exp\left(-\dfrac{r^2}{r_0^2}\dfrac{\tau}{\tau+t-t_0}\right)\right\},\quad t>t_0.\end{cases}\tag{9}$$

The temperature on the axis of the beam will be

$$T(0,\ t)=\begin{cases}0, \quad t<0,\\ \dfrac{4kI_0\tau^3}{c\rho t_0^2}\left[\left(1+\dfrac{t}{\tau}\right)\left(1+\dfrac{t-t_0}{\tau}\right)\ln\left(1+\dfrac{t}{\tau}\right)-\dfrac{t}{\tau}\times\right.\\ \qquad\left.\times\left(1+\dfrac{3t-2t_0}{\tau}\right)\right],\quad t_0\geqslant t\geqslant 0,\\ \dfrac{4kI_0\tau^3}{c\rho t_0^2}\left[\left(1+\dfrac{t}{\tau}\right)\left(1+\dfrac{t-t_0}{\tau}\right)\ln\dfrac{\tau+t}{\tau+t-t_0}-\dfrac{t_0}{\tau}\times\right.\\ \qquad\left.\times\left(1+\dfrac{2t-t_0}{\tau}\right)\right],\quad t>t_0.\end{cases} \tag{10}$$

d) The intensity of the laser emission varies exponentially with time:

$$I(t)=\begin{cases}0, \quad t<0,\\ I_0\exp(-\beta t),\quad t\geqslant 0.\end{cases}$$

The temperature on the axis of the beam varies as given by

$$T(0,\ t)=\begin{cases}0, \quad t<0,\\ \dfrac{kI_0\tau}{c\rho}\{\mathrm{Ei}[-\beta(\tau+t)]-\mathrm{Ei}(-\beta\tau)\}\times\\ \qquad\times\exp[-\beta(\tau+t)],\quad t\geqslant 0.\end{cases} \tag{11}$$

The time behavior of the intensity of the emission of a pulsed laser can often be approximated by the difference between two exponential relations. The temperature on the axis of the beam in this case is given by the difference between expressions of the form (11).

The temperature field induced by laser emission pulses which repeat periodically after a time interval t_n is given by the sum of the temperature fields induced by each of the radiation pulses:

$$T(r,\ t)=\sum_{i=0}^{m}T(r,\ t-it_n).$$

Here m is the number of laser pulses (i.e., the integer part of t/t_n), and $T(r,\ t-it_n)$ is the temperature field produced by the i-th laser pulse.

Discussion of the Results Obtained

The expressions given above describe the temperature field during the laser pulse and after the pulse is completed, and they take into account the change in the laser emission intensity with time. The values obtained for the temperature field are more general than those obtained previously in [1-3], where for a Gaussian beam the temperature field was considered during the laser pulse assuming constant intensity. However, the expressions obtained for the temperature field have the complex form given by Eqs. (5)-(11). These expressions can be simplified in some important practical cases: a) for times considerably less than the characteristic temperature relaxation time ($t \ll \tau$); b) for times considerably greater than the length of the laser pulse ($t \gg t_0$). In order to obtain asymptotic expressions for the temperature field, the general solution (3), using Eq. (4), can be expanded in a Taylor series. Assuming that the first two terms are not equal to zero in the expansion, the time interval for which the temperature field can be described by the first term of the expansion with an accuracy η (for example, $\eta = 0.1$) can be obtained. The temperature field can then be described by the following asymptotic expressions:

$$T(r,\ t)=\begin{cases}\dfrac{k_e(t)}{c\rho}\exp\left(-\dfrac{r^2}{r_0^2}\right),\quad t<2\eta\tau,\\ \dfrac{k_e(t_0)}{c\rho}\cdot\dfrac{1}{1+\dfrac{t}{\tau}}\exp\left(-\dfrac{r^2}{r_0^2}\cdot\dfrac{1}{1+\dfrac{t}{\tau}}\right),\quad t_0<\eta(t+\tau),\end{cases} \tag{12}$$

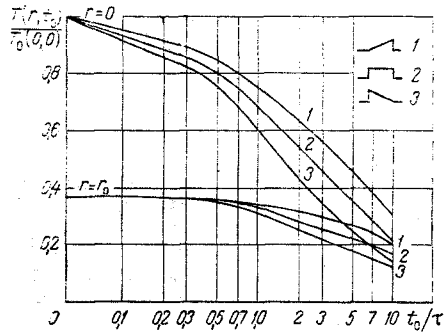

Fig. 1. Dependence of the temperature on the thermal diffusivity for different time dependences (1-3) of the laser emission pulse.

where $e(t) = \int_0^t I(t)dt$ is the energy density of the laser emission on the axis of the beam. It is easy to show that for times $t < 2\eta\tau$ with an error less than $\eta \cdot 100\%$ the thermal conductivity of the medium can be neglected. For fairly long times $[t > (t_0/\eta) - \tau]$ the temperature field is identical with the relaxation temperature field of an instantaneously heated region (4).

The applicability of the asymptotic expressions for the temperature field (12) is determined by the characteristic thermal-diffusivity time τ. In focused laser beams (for $r_0 = 50\ \mu$), operating under free-running conditions, for K, BK, LK, F, and TF optical glasses [4], the characteristic thermal-diffusivity time is 1-2 nsec. To determine the temperature with an accuracy of 10% in glasses the thermal conductivity must be taken into account for pulse lengths of 0.2-0.4 nsec, and in optical semiconductors (GaAs and Ge) and dielectric media with good thermal conductivity ($Al_2O_3:Cr^{3+}$, $LiNbO_3$), for pulse lengths $\sim 10^{-6}$ sec. Consequently, it is necessary to take the thermal conductivity of the medium into account in calculations of the thermoelastic stresses and in measurements of the optical constants of the medium by the interference method [5].

The effect of the laser pulse shape and the thermal diffusivity of the medium on the temperature field can be followed if the intensity of the laser emission is normalized in such a way that the energy of the radiation in the pulse is the same. Figure 1 shows the dependence of the temperature of the medium at the end of the radiation on the characteristic thermal-diffusivity time for laser intensities which are linearly increasing with time, independent of time, and linearly decreasing with time, for two characteristic points in the specimen — on the axis of the beam ($r = 0$) and at a distance from the axis of the beam ($r = r_0$). It is seen from the figure that when $t_0/\tau \approx 1$ the temperature fields differ by up to 20%. The calculations show that on the axis of the beam for "parabolic" and "rectangular" pulses ($t_0/\tau = 10$ and $t_0/\tau = 1$) the temperatures at the end of the pulse differ by 20% and 2%, respectively. By measuring the temperature field as a function of the pulse shape one can approximate the actual pulse shape by means of the relations given above for the intensity of the laser emission as a function of time, and using the above solutions, one can calculate the temperature field with any required accuracy.

The solutions (5)-(11) obtained can be employed to determine the temperature field at any instant of time produced by absorption of a single pulse or repeated pulses of laser emission having a Gaussian spatial distribution, taking into account the variation of its intensity with time.

NOTATION

k, linear light absorption coefficient; l, sample length; r, distance from the axis; r_0, radius of the Gaussian beam; t, instantaneous time; t_0, pulse lengths; τ, characteristic time of departure of heat; c, heat capacity; ρ, density; q, volume source of heat; I(r, t), laser emission intensity; f, spatial profile of the pulse; I(t), pulse time shape; e, energy density; η, the desired relative error; λ, the thermal conductivity; t_n, pulse repetition time.

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A SIMPLE PROCEDURE FOR CONSTRUCTING SOLUTIONS OF NONLINEAR HEAT-CONDUCTION PROBLEMS BY THE KANTOROVICH METHOD

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A simplified procedure based on expansion in the neighborhood of an approximate solution is discussed for solution of the quasilinear heat-conduction equation.

It is generally known that either the energy method or the more promising method of Galerkin [2] is used in connection with the method of Kantorovich [1]. The crux of either approach is that in the solution of nonlinear problems of mathematical physics one must inevitably cope with systems of nonlinear ordinary differential equations and algebraic equations, a prospect that often incurs insurmountable difficulties and naturally imposes limitations on practical applications. A vital problem in this connection is the search for a procedure that can be used to construct solutions of nonlinear problems by reduction to ordinary differential equations without having to solve systems of nonlinear equations, at least in the stage of refinement of the initial approximation.

Below we consider such a procedure for the quasilinear heat-conduction equation in three-dimensional space and for a general type of nonlinear boundary-value problem. In addition to the requirements of existence and uniqueness of a solution, we impose constraints that are quite strong, but are nonetheless frequently justified, as a rule, in a number of practical problems, as for example in the area of heat physics: 1) The solution T(x, y, z, t) is representable with sufficient practical accuracy in some neighborhood of a certain initial approximation $T = T_0(x, y, z, t)$ by an equation in the form of a power series, finite or infinite, which is differentiable with respect to the coordinates and time; 2) in the neighborhood of $T = T_0(x, y, z, t)$ the coefficients in the equation and in the boundary conditions are analytical functions of T.

Consider the equation

$$f_1(T)\,\frac{\partial T}{\partial t} = \nabla\,[f_2(T)\,\nabla T] \tag{1}$$

subject to the boundary conditions on the surface s

$$f_3(T)\,\nabla T + f_4(T)\,|_s = 0. \tag{2}$$

In accordance with constraints 1) and 2) we represent T and the functions f_i (i = 1, 2, 3, 4) in the form

$$T = \alpha_0 + \varepsilon\alpha_1 + \ldots \varepsilon^n\alpha_n, \tag{3}$$

$$f_i = f_i|_{T=T_0} + \left.\frac{\partial f_i}{\partial T}\right|_{T=T_0}(T-T_0) + \ldots = \beta_{ki}\,(\varepsilon^m\alpha_m)^k. \tag{4}$$

The "Einstein rule," i.e., summation with respect to a certain index, is tacitly understood at all times.

As the initial expression for α_0 we can take the solution given by, for example, the integral [3, 4] or any other suitable method.

We substitute (3) and (4) into (1) and (2). We obtain

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